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Hybrid High-Order Methods for Variable-Diffusion Problems on General Meshes

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Abstract

We extend the Hybrid High-Order method introduced by the authors for the Poisson problem to problems with heterogeneous/anisotropic diffusion. The cornerstone is a local discrete gradient reconstruction from element- and face-based polynomial degrees of freedom. Optimal error estimates are proved.

Résumé

Méthodes hybrides d'ordre élevé pour des problèmes à diffusion variable sur des maillages généraux. Nous étendons la méthode hybride d'ordre élevé conçue par les auteurs pour le problème de Poisson à des problèmes de diffusion hétérogène/anisotrope. La pierre angulaire est une reconstruction locale du gradient discret à partir des degrés de liberté polynomiaux sur les éléments et les faces. On établit des estimations d'erreur optimales.

1. Introduction

Let $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$, denote an open, bounded, polytopic domain. Let $f \in L^2(\Omega)$ and, for a subset $X \subset \overline{\Omega}$, denote by $(\cdot, \cdot)_X$ and $\|\cdot\|_X$ the inner product and norm in $L^2(X)$, respectively. We focus on the following variable-diffusion problem: Find $u \in U_0 := H_0^1(\Omega)$ such that

$$(\kappa \nabla u, \nabla v)_\Omega = (f, v)_\Omega \quad \forall v \in U_0, \quad (1)$$

where κ is a bounded, tensor-valued function in Ω , taking symmetric values with lowest eigenvalue uniformly bounded from below away from zero. Owing to the Lax–Milgram Lemma, problem (1) is well-posed.

The approximation of diffusive problems on general polytopic meshes has received an increasing attention lately. Several low-order methods have been developed; see, e.g., [1, 2] and references therein. Recently, high-order methods have also become available; we mention the high-order Mimetic Finite Difference (MFD) schemes [3, 4], the Virtual Element Method [5], the Mixed High-Order method [6], and the Hybrid High-Order (HHO) methods [7, 8]. For the latter, the degrees of freedom (DOFs) are scalar-valued polynomials at mesh elements and faces up to some degree $k \geq 0$ (as for the MFD schemes in [4]), and the construction hinges on (i) a local discrete gradient reconstruction of order k and (ii) a least-squares local penalty that weakly enforces the matching between element- and face-based DOFs while preserving the order of the gradient reconstruction. This design leads to optimal energy- and L^2 -norm error estimates; cf. [7] for the Poisson problem (κ being the identity tensor in (1)) and [8] for (quasi-incompressible) linear elasticity.

The purpose of the present work is to extend the HHO method of [7] to the variable-diffusion problem (1). The key idea is to modify the gradient reconstruction so as to account for the diffusion tensor κ . Then, adapting the ideas of [7], we prove stability of the discrete problem and derive optimal error estimates. We make the reasonable assumption that there is a partition P_Ω of Ω so that κ is piecewise Lipschitz. For simplicity of exposition, we also assume that κ is a piecewise polynomial; otherwise, an additional quadrature error has to be accounted for. In applications from the geosciences, κ can often be taken piecewise constant.

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2. Discrete setting and local gradient reconstruction

We consider admissible mesh sequences in the sense of [9, Sect. 1.4]. Each mesh \mathcal{T}_h in the sequence is a finite collection $\{T\}$ of nonempty, disjoint, open, polytopic elements such that $\bar{\Omega} = \bigcup_{T \in \mathcal{T}_h} \bar{T}$ and $h = \max_{T \in \mathcal{T}_h} h_T$ (with h_T the diameter of T), and there is a matching simplicial submesh of \mathcal{T}_h with locally equivalent mesh size and which is shape-regular in the usual sense. For all $T \in \mathcal{T}_h$, the faces of T are collected in the set \mathcal{F}_T . In an admissible mesh sequence, $\text{card}(\mathcal{F}_T)$ is uniformly bounded, the usual discrete and multiplicative trace inequalities hold on element faces, and the L^2 -orthogonal projector onto polynomial spaces enjoys optimal approximation properties on each mesh element. Let a polynomial degree $k \geq 0$ be fixed. For all $T \in \mathcal{T}_h$, we define the local space of DOFs as $\mathbf{U}_T^k := \mathbb{P}_d^k(T) \times \{\times_{F \in \mathcal{F}_T} \mathbb{P}_{d-1}^k(F)\}$, where $\mathbb{P}_d^k(T)$ (resp., $\mathbb{P}_{d-1}^k(F)$) is spanned by the restrictions to T (resp., F) of d -variate (resp., $(d-1)$ -variate) polynomials of total degree $\leq k$. In what follows, $A \lesssim B$ denotes the inequality $A \leq CB$ with positive constant C independent of the polynomial degree k , the meshsize h , and the diffusion tensor κ . We assume that each mesh \mathcal{T}_h in the sequence is compatible with the partition P_Ω associated with the diffusion tensor. We denote by κ_T^b and κ_T^\sharp the lowest and largest eigenvalue of κ in T , respectively, and we introduce the local heterogeneity/anisotropy ratio $\rho_T := \kappa_T^\sharp / \kappa_T^b \geq 1$. In what follows, we explicitly track the dependency of the bounds on the ratio ρ_T . To avoid the proliferation of symbols, we assume that for all $T \in \mathcal{T}_h$, the Lipschitz constant of κ in T , say L_T^κ , satisfies $L_T^\kappa \lesssim \kappa_T^\sharp$.

For all $T \in \mathcal{T}_h$, we define the local gradient reconstruction operator $\mathbf{G}_T^k : \mathbf{U}_T^k \rightarrow \nabla \mathbb{P}_d^{k+1}(T)$ such that, for all $\mathbf{v} := (\mathbf{v}_T, (\mathbf{v}_F)_{F \in \mathcal{F}_T}) \in \mathbf{U}_T^k$ and all $w \in \mathbb{P}_d^{k+1}(T)$,

$$(\kappa \mathbf{G}_T^k \mathbf{v}, \nabla w)_T = (\kappa \nabla \mathbf{v}_T, \nabla w)_T + \sum_{F \in \mathcal{F}_T} (\mathbf{v}_F - \mathbf{v}_T, \nabla w \cdot \kappa \cdot \mathbf{n}_{TF})_F, \quad (2)$$

which can be computed by solving a local (well-posed) Neumann problem in $\mathbb{P}_d^{k+1}(T)$. We next introduce the potential reconstruction operator $p_T^k : \mathbf{U}_T^k \rightarrow \mathbb{P}_d^{k+1}(T)$ such that, for all $\mathbf{v} \in \mathbf{U}_T^k$, $\nabla p_T^k \mathbf{v} := \mathbf{G}_T^k \mathbf{v}$ and $\int_T p_T^k \mathbf{v} := \int_T \mathbf{v}_T$ ($p_T^k \mathbf{v}$ is well-defined since $\mathbf{G}_T^k \mathbf{v} \in \nabla \mathbb{P}_d^{k+1}(T)$). Finally, we define the local interpolation operator $\mathbf{l}_T^k : H^1(T) \rightarrow \mathbf{U}_T^k$ such that, for all $v \in H^1(T)$, $\mathbf{l}_T^k v := (\pi_T^k v, (\pi_F^k v)_{F \in \mathcal{F}_T})$, where π_T^k and π_F^k are the L^2 -orthogonal projectors onto $\mathbb{P}_d^k(T)$ and $\mathbb{P}_{d-1}^k(F)$, respectively.

Lemma 2.1 (Approximation properties for $p_T^k \mathbf{l}_T^k$). *The following holds for all $v \in H^{k+2}(T)$ with $\alpha = 1/2$ if κ is piecewise constant and $\alpha = 1$ in the general case:*

$$\|v - p_T^k \mathbf{l}_T^k v\|_T + h_T^{1/2} \|v - p_T^k \mathbf{l}_T^k v\|_{\partial T} + h_T \|\nabla(v - p_T^k \mathbf{l}_T^k v)\|_T + h_T^{3/2} \|\nabla(v - p_T^k \mathbf{l}_T^k v)\|_{\partial T} \lesssim \rho_T^\alpha h_T^{k+2} \|v\|_{H^{k+2}(T)}. \quad (3)$$

Proof. Let $v \in H^{k+2}(T)$. A direct calculation using (2), the definitions of p_T^k and \mathbf{l}_T^k , and integration by parts shows that, for all $w \in \mathbb{P}_d^{k+1}(T)$,

$$(\kappa \nabla(v - p_T^k \mathbf{l}_T^k v), \nabla w)_T = ((\kappa - \bar{\kappa}_T) \nabla(v - \pi_T^k v), \nabla w)_T - \sum_{F \in \mathcal{F}_T} (\pi_F^k v - \pi_T^k v, \nabla w \cdot (\kappa - \bar{\kappa}_T) \cdot \mathbf{n}_{TF})_F,$$

where $\bar{\kappa}_T$ denotes the mean-value of κ in T . Note that the right-hand side vanishes if κ is piecewise constant. In the general case, owing to the assumptions on κ and using the approximation properties of the L^2 -orthogonal projectors along with a discrete trace inequality for $\|\kappa^{1/2} \nabla w\|_F$, we infer that

$$|(\kappa \nabla(v - p_T^k \mathbf{l}_T^k v), \nabla w)_T| \lesssim L_T^\kappa h_T h_T^k \|v\|_{H^{k+1}(T)} \|\nabla w\|_T \lesssim \kappa_T^\sharp h_T^{k+1} \|v\|_{H^{k+1}(T)} \|\nabla w\|_T. \quad (4)$$

We now observe that

$$\|\kappa^{1/2} \nabla(v - p_T^k \mathbf{l}_T^k v)\|_T^2 = (\kappa \nabla(v - p_T^k \mathbf{l}_T^k v), \nabla(v - \pi_T^{k+1} v))_T + (\kappa \nabla(v - p_T^k \mathbf{l}_T^k v), \nabla(\pi_T^{k+1} v - p_T^k \mathbf{l}_T^k v))_T. \quad (5)$$

Denote by \mathfrak{I}_1 and \mathfrak{I}_2 the addends on the right-hand side of (5). Using the Cauchy-Schwarz inequality and the approximation properties of π_T^{k+1} , we obtain $|\mathfrak{I}_1| \lesssim \|\kappa^{1/2} \nabla(v - p_T^k \mathbf{l}_T^k v)\|_T (\kappa_T^\sharp)^{1/2} h_T^{k+1} \|v\|_{H^{k+2}(T)}$.

When κ is piecewise constant, \mathfrak{I}_2 vanishes, so that using Young's inequality yields $\|\nabla(v - p_T^k l_T^k v)\|_T \leq (\kappa_T^b)^{-1/2} \|\kappa^{1/2} \nabla(v - p_T^k l_T^k v)\|_T \lesssim \rho_T^{1/2} h_T^{k+1} \|v\|_{H^{k+2}(T)}$. In the general case, using (4) with $w = (\pi_T^{k+1} v - p_T^k l_T^k v)$ and since $\|\nabla(\pi_T^{k+1} v - p_T^k l_T^k v)\|_T = \|\nabla \pi_T^{k+1}(v - p_T^k l_T^k v)\|_T \lesssim \|\nabla(v - p_T^k l_T^k v)\|_T$ owing to the H^1 -stability of the projector π_T^{k+1} , we infer that $|\mathfrak{I}_2| \lesssim \rho_T^{1/2} (\kappa_T^\#)^{1/2} h_T^{k+1} \|v\|_{H^{k+1}(T)} \|\kappa^{1/2} \nabla(v - p_T^k l_T^k v)\|_T$, which leads to the estimate on $\|\nabla(v - p_T^k l_T^k v)\|_T$ in (3). The other terms in (3) are then bounded as in [7, Lemma 3]. \square

Remark 1 ($\alpha = 0$). It is also possible to take $\alpha = 0$ whenever, for all $T \in \mathcal{T}_h$, the eigenvectors of $\kappa|_T$ are constant and its eigenvalues satisfy, with obvious notation, $|\lambda(x) - \bar{\lambda}_T| \lesssim h_T \lambda(x)$ for all $x \in T$.

3. Discrete problem and stability

For all $T \in \mathcal{T}_h$, we introduce the local bilinear forms a_T and s_T on $\mathbf{U}_T^k \times \mathbf{U}_T^k$ such that

$$a_T(\mathbf{u}, \mathbf{v}) := (\kappa \mathbf{G}_T^k \mathbf{u}, \mathbf{G}_T^k \mathbf{v})_T + s_T(\mathbf{u}, \mathbf{v}), \quad s_T(\mathbf{u}, \mathbf{v}) := \sum_{F \in \mathcal{F}_T} \frac{\kappa_F}{h_F} (\pi_F^k(\mathbf{u}_F - P_T^k \mathbf{u}), \pi_F^k(\mathbf{v}_F - P_T^k \mathbf{v}))_F, \quad (6)$$

with $\kappa_F := \|\mathbf{n}_{TF} \cdot \kappa \cdot \mathbf{n}_{TF}\|_{L^\infty(F)}$ and the local potential reconstruction $P_T^k : \mathbf{U}_T^k \rightarrow \mathbb{P}_d^{k+1}(T)$ such that $P_T^k \mathbf{v} := \mathbf{v}_T + (p_T^k \mathbf{v} - \pi_T^k p_T^k \mathbf{v})$. We define the global space of DOFs by patching interface values, so that $\mathbf{U}_h^k := \{\times_{T \in \mathcal{T}_h} \mathbb{P}_d^k(T)\} \times \{\times_{F \in \mathcal{F}_h} \mathbb{P}_{d-1}^k(F)\}$, and, for all $T \in \mathcal{T}_h$, we denote by $\mathbf{L}_T : \mathbf{U}_h^k \rightarrow \mathbf{U}_T^k$ the restriction operator that maps the global DOFs in \mathbf{U}_h^k to the corresponding local DOFs in \mathbf{U}_T^k . The discrete problem consists in seeking $\mathbf{u}_h \in \mathbf{U}_{h,0}^k := \{\mathbf{v}_h = ((\mathbf{v}_T)_{T \in \mathcal{T}_h}, (\mathbf{v}_F)_{F \in \mathcal{F}_h}) \in \mathbf{U}_h^k \mid \mathbf{v}_F \equiv 0 \forall F \in \mathcal{F}_h^b\}$ such that

$$a_h(\mathbf{u}_h, \mathbf{v}_h) := \sum_{T \in \mathcal{T}_h} a_T(\mathbf{L}_T \mathbf{u}_h, \mathbf{L}_T \mathbf{v}_h) = \sum_{T \in \mathcal{T}_h} (f, \mathbf{v}_T)_T =: l_h(\mathbf{v}_h) \quad \forall \mathbf{v}_h \in \mathbf{U}_{h,0}^k. \quad (7)$$

To analyze the stability of the discrete problem, we introduce the following seminorm on \mathbf{U}_T^k :

$$\|\mathbf{v}\|_{\kappa,T}^2 := \|\kappa^{1/2} \nabla \mathbf{v}_T\|_T^2 + \sum_{F \in \mathcal{F}_T} \frac{\kappa_F}{h_F} \|\mathbf{v}_F - \mathbf{v}_T\|_F^2, \quad (8)$$

and we set $\|\mathbf{v}_h\|_{\kappa,h}^2 := \sum_{T \in \mathcal{T}_h} \rho_T^{-1} \|\mathbf{L}_T \mathbf{v}_h\|_{\kappa,T}^2$ for all $\mathbf{v}_h \in \mathbf{U}_h^k$. Observe that $\|\cdot\|_{\kappa,h}$ is a norm on $\mathbf{U}_{h,0}^k$.

Lemma 3.1 (Stability). *The following inequalities hold for all $\mathbf{v} \in \mathbf{U}_T^k$:*

$$\rho_T^{-1} \|\mathbf{v}\|_{\kappa,T}^2 \lesssim a_T(\mathbf{v}, \mathbf{v}) \lesssim \rho_T \|\mathbf{v}\|_{\kappa,T}^2. \quad (9)$$

Consequently, $\|\mathbf{v}_h\|_{\kappa,h}^2 \lesssim a_h(\mathbf{v}_h, \mathbf{v}_h)$ for all $\mathbf{v}_h \in \mathbf{U}_h^k$ and problem (7) is well-posed.

Proof. We adapt the proof of [7, Lemma 4]. Concerning the face terms, we obtain

$$\sum_{F \in \mathcal{F}_T} \frac{\kappa_F}{h_F} \|\mathbf{v}_F - \mathbf{v}_T\|_F^2 \leq s_T(\mathbf{v}, \mathbf{v}) + \rho_T \|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T^2, \quad s_T(\mathbf{v}, \mathbf{v}) \lesssim \sum_{F \in \mathcal{F}_T} \frac{\kappa_F}{h_F} \|\mathbf{v}_F - \mathbf{v}_T\|_F^2 + \rho_T \|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T^2. \quad (10)$$

To compare $\|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T$ and $\|\kappa^{1/2} \nabla \mathbf{v}_T\|_T$, we observe that, for all $w \in \mathbb{P}_d^{k+1}(T)$ and all $F \in \mathcal{F}_T$,

$$\|\nabla w \cdot \kappa \cdot \mathbf{n}_{TF}\|_F^2 \leq (|\mathbf{n}_{TF} \cdot \kappa \cdot \mathbf{n}_{TF}|, |\nabla w \cdot \kappa \cdot \nabla w|)_F \lesssim \frac{\kappa_F}{h_F} \|\kappa^{1/2} \nabla w\|_T^2, \quad (11)$$

where we have used the Cauchy-Schwarz inequality for κ , the definition of κ_F , and a discrete trace inequality. Taking $w = \mathbf{v}_T$ in the definition (2) of $\mathbf{G}_T^k \mathbf{v}$ yields $\|\kappa^{1/2} \nabla \mathbf{v}_T\|_T^2 = (\kappa \mathbf{G}_T^k \mathbf{v}, \nabla \mathbf{v}_T)_T - \sum_{F \in \mathcal{F}_T} (\mathbf{v}_F - \mathbf{v}_T, \nabla \mathbf{v}_T \cdot \kappa \cdot \mathbf{n}_{TF})_F$. Hence, using (11), a discrete trace inequality for $\|\kappa^{1/2} \nabla \mathbf{v}_T\|_F$, the first bound in (10), $\rho_T \geq 1$, and Young's inequality yields

$$\|\kappa^{1/2} \nabla \mathbf{v}_T\|_T^2 \lesssim \|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T^2 + \sum_{F \in \mathcal{F}_T} \frac{\kappa_F}{h_F} \|\mathbf{v}_F - \mathbf{v}_T\|_F^2 \lesssim \rho_T \|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T^2 + s_T(\mathbf{v}, \mathbf{v}).$$

Moreover, since $\|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T = \sup_{w \in \mathbb{P}_d^{k+1}(T)} \frac{(\kappa \mathbf{G}_T^k \mathbf{v}, \nabla w)_T}{\|\kappa^{1/2} \nabla w\|_T}$ and proceeding similarly leads to $\|\kappa^{1/2} \mathbf{G}_T^k \mathbf{v}\|_T \lesssim \|\mathbf{v}\|_{\kappa,T}$. Combining the above bounds yields (9), and the rest of the proof is straightforward. \square

4. Error analysis

Theorem 4.1 (Energy-error estimate). *Let $u \in U_0$ solve (1) and let $u_h \in U_{h,0}^k$ solve (7). Assume that $u|_T \in H^{k+2}(T)$ for all $T \in \mathcal{T}_h$. Then, letting $\hat{u}_h := ((\pi_T^k u)_{T \in \mathcal{T}_h}, (\pi_F^k u)_{F \in \mathcal{F}_h}) \in U_{h,0}^k$ and, recalling the definition of α from Lemma 2.1, the following holds with consistency error $\mathcal{E}_h(v_h) := a_h(\hat{u}_h, v_h) - l_h(v_h)$:*

$$\|\hat{u}_h - u_h\|_{\kappa,h} \lesssim \sup_{v_h \in U_{h,0}^k, \|v_h\|_{\kappa,h}=1} \mathcal{E}_h(v_h) \lesssim \left\{ \sum_{T \in \mathcal{T}_h} \kappa_T^\# \rho_T^{1+2\alpha} h_T^{2(k+1)} \|u\|_{H^{k+2}(T)}^2 \right\}^{1/2}. \quad (12)$$

Proof. We adapt the proof of [7, Theorem 8]. The first inequality in (12) is an immediate consequence of Lemma 3.1. Proceeding as in [7] with $\check{u}_T := p_T^k L_T \hat{u}_h = p_T^k l_T^k(u|_T)$ and $v_h \in U_{h,0}^k$ with $\|v_h\|_{\kappa,h} = 1$ leads to

$$\mathcal{E}_h(v_h) = \sum_{T \in \mathcal{T}_h} (\kappa \nabla(\check{u}_T - u), \nabla v_T)_T + \sum_{T \in \mathcal{T}_h} \sum_{F \in \mathcal{F}_T} (v_F - v_T, (\nabla \check{u}_T - \nabla u) \cdot \kappa \cdot n_{TF})_F + \sum_{T \in \mathcal{T}_h} s_T(L_T \hat{u}_h, L_T v_h).$$

Denote by $\mathfrak{T}_1, \mathfrak{T}_2, \mathfrak{T}_3$ the three terms on the right-hand side. Combining the results of Lemmas 2.1 and 3.1, we infer that $|\mathfrak{T}_1 + \mathfrak{T}_2|^2 \lesssim \sum_{T \in \mathcal{T}_h} \kappa_T^\# \rho_T^{1+2\alpha} h_T^{2(k+1)} \|u\|_{H^{k+2}(T)}^2$. Moreover, since $s_T(L_T \hat{u}_h, L_T v_h) \leq s_T(L_T \hat{u}_h, L_T \hat{u}_h)^{1/2} s_T(L_T v_h, L_T v_h)^{1/2}$, proceeding as in [7] for the first factor, and using the second bound in (10) for the second factor yields $|\mathfrak{T}_3|^2 \lesssim \sum_{T \in \mathcal{T}_h} \kappa_T^\# \rho_T^{1+2\alpha} h_T^{2(k+1)} \|u\|_{H^{k+2}(T)}^2$. \square

Finally, adapting the proof of [7, Theorem 10] leads to the following L^2 -norm error estimate.

Theorem 4.2 (L^2 -error estimate). *Assume elliptic regularity for problem (1) in the form $\|z\|_{H^2(\Omega)} \lesssim \|g\|_\Omega$ for all $g \in L^2(\Omega)$ and $z \in U_0$ solving (1) with data g . Assume $f \in H^{k+\delta}(\Omega)$ with $\delta = 0$ for $k \geq 1$ and $\delta = 1$ for $k = 0$. Then, using the same notation as in Theorem 4.1, and defining the piecewise polynomial functions \hat{u}_h and u_h such that $\hat{u}_h|_T = \pi_T^k u$ and $u_h|_T = u_T$ for all $T \in \mathcal{T}_h$, the following holds:*

$$\|\hat{u}_h - u_h\|_\Omega \lesssim |(\kappa^\#)^{1/2} \rho^{1/2+\alpha} h|_{\ell^\infty} \left\{ \sum_{T \in \mathcal{T}_h} \kappa_T^\# \rho_T^{1+2\alpha} h_T^{2(k+1)} \|u\|_{H^{k+2}(T)}^2 \right\}^{1/2} + h^{k+2} \|f\|_{H^{k+\delta}(\Omega)},$$

where $|(\kappa^\#)^{1/2} \rho^{1/2+\alpha} h|_{\ell^\infty} := \max_{T \in \mathcal{T}_h} (\kappa_T^\#)^{1/2} \rho_T^{1/2+\alpha} h_T$.

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